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Rhetorical Pattern Finding

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Abstract

In this paper, we research rhetorical patterns from a musicological and computational standpoint. First, a theoretical examination of what constitutes a rhetorical pattern is conducted. Out of that examination, which includes primary sources and the study of the main composers, a formal definition of rhetorical patterns is proposed. Among the rhetorical figures, a set of imitative rhetorical figures is selected for our study, namely, epizeuxis, palilogy, synonymia, and polyptoton. Next, we design a computational model of the selected rhetorical patterns to automatically find those patterns in a corpus consisting of masses by Renaissance composer Tomás Luis de Victoria. In order to have a ground truth with which to test out our model, a group of experts manually annotated the rhetorical patterns. To deal with the problem of reaching a consensus on the annotations, a four-round Delphi method was followed by the annotators. The rhetorical patterns found by the annotators and by the algorithm are compared and their differences discussed. The algorithm reports almost all the patterns annotated by the experts and some additional patterns. The algorithm reports almost all the patterns annotated by the experts (recall: 98.11%) and some additional patterns (precision: 71.73%). These patterns correspond to rhetorical patterns within other rhetorical patterns, which were overlooked by the annotators on the basis of their contextual knowledge. These results pose issues as to how to integrate that contextual knowledge into the computational model.

Keywords

Computational Models for Rhetorical Patterns, Delphi Method, Imitative Patterns, Musicological Models for Rhetorical Patterns, Musical Patterns, Pattern Annotation, Pattern Finding Algorithms, Renaissance Music, Rhetorical Patterns, Victoria's Masses.

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I. Introduction

In this work, we study rhetorical patterns, in particular rhetorical patterns by repetition, from a musicological and computational standpoint. This kind of pattern is associated to highly imitative music of contrapuntal nature in the Renaissance and Baroque eras. To the best of our knowledge, rhetorical patterns have received very little attention in the modern musicological literature. The contribution of this research follows a path that starts by establishing a conceptual framework for the definition and characterization of rhetorical patterns, which is a question in need of further research in musicology; see Section II. After that, issues have arisen when annotating these patterns —including the problem of subjectivity and consensus-reaching—, are examined. Manual annotations are all at the same time valuable, cognitively demanding, time-consuming, and prone to error. Furthermore, when a corpus of music is relatively large, manually detecting all the patterns is not feasible and is often unrealistic. Therefore, in this study, we also took into account those patterns that the experts overlooked in their annotations. In our case, manual annotations by experts were needed to understand better the nature of rhetorical patterns as well as to provide ground truth to test the pattern-discovery algorithm. Moreover, when a set of experts annotate some music, the problem of reaching consensus among them soon arises. In order to tackle this problem, a consensus-reaching scheme, called Delphi, was implemented. Both the issues of manual annotations and consensus-reaching are addressed in Section III.A. We also built a computational models for the rhetorical patterns, which included formal computational definitions of the rhetorical patterns (Section II.B). To automatically extract and mine rhetorical patterns, we used an existing pattern-finding algorithm, the BIDE algorithm. We then proceeded to test our method on the work of Tomás Luis de Victoria (details in Section II.C) by determining whether the category of a discovered pattern matches the category assigned by annotators. A classification task —whether a discovered pattern is an annotated pattern—was set up and performance measures were computed and analyzed (Section IV). It turned out that the algorithm predicted the correct category for most of the annotated patterns (recall: 98.11%), but interestingly enough, it also found additional patterns that were overlooked by the experts (precision: 71.73%). Lastly, the paper comes to an end where conclusions about the results obtained in this research are discussed.

Musicologists and other musical scholars can benefit from this work in many ways. First, we proposed a conceptual framework for rhetorical patterns. That framework unifies different definitions given in the literature (see Section II.B) and proposes a rigorous definition of the main rhetorical patterns found in the Renaissance and Baroque music. Second, to have the possibility of searching for rhetorical patterns in large corpora of music is highly advantageous. Manual annotation of patterns is a prone-to-error and tedious task. As we will see in this paper, algorithms can detect some patterns that are diÿcult to detect by the experts. Third, we made use of a novel method to

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reach consensus among experts, the Delphi method. To the best of our knowledge, such method have hardly been used in research involving consensus-reaching in musicology. Finally, the results obtained in this work open new venues for further research, especially the sub-pattern problem (the problem of having nested patterns; see Section II.B).

II. Rhetorical Patterns in Music

A. Musical Patterns

Patterns provide the musical discourse with both meaning and structure. In terms of meaning, they bear a resemblance to phonemes, the smallest units of speech that perceptually distinguish one word from another, as patterns can broadly be defined as the shortest meaningful sequences in a piece of music. Indeed, patterns carry enough relevant information to create musical statements or advance musical arguments on their own. In terms of structure, through purposeful repetition, they help construct musical syntax throughout the piece. Surprisingly enough, an agreed-upon definition of musical patterns has proved elusive. Some authors define musical patterns as sequences of notes that are repeated at least twice in the piece; for example, that is the case of the MIREX 2013 Repeated Themes & Sections task [1]; see also [2]. However, from a conceptual and musical standpoint this definition seems to be somewhat insuÿcient. Much research into musical pattern finding merely relies on repetition to find patterns in musical corpora. The rationale behind that strategy is that if a sequence of musical events is repeated, it should be because such sequence has musical entity and therefore can be classed as a musical pattern. Many pattern-finding algorithms limit themselves to track down the most frequent sequences in the music without examining the qualities of the sequence or where their occurrences appear in the piece. However, not every repeated sequence of musical events is necessarily a pattern. The famous opening theme in the first movement of Tchaikovsky's first Piano Concerto is clearly a pattern, which is remembered by all attentive listeners, but is never repeated again and certainly is not developed. It simply stands by itself as a flamboyant statement bearing no relationship to the rest of the movement. Contrariwise, the initial pattern in the first movement of Beethoven's Fifth Symphony is the backbone of the whole movement, where that pattern is developed and transformed ad infinitum.

What is then a musical pattern? Other authors ([3], [4]) use more or less vague definitions such as a musical patterns are "perceptible repetitions in a musical piece," or "an excerpt of special importance," or "a salient fragment," or "a prominent unit." For a sequence to be considered a musical pattern it needs to possess some extra attributes. A musical pattern is here defined as a musical event that constitutes a musical whole and is repeated as to create structure in a musical piece. Gestalt theory can shed light over the precise mechanisms underlying the formation of musical patterns through rules of proximity, similarity, and good continuation. For example, Deutsch [5] describes several perceptual mechanisms operating in the construction of a musical pattern as a musical whole, such as grouping of musical sounds —grouping by pitch proximity, grouping by timbre, grouping by temporal proximity, among others—, whereas Lartillot [6] identifies musical patterns in terms of style-based groupings, local boundaries, and repetition (his work in turn builds on the generative theory of Lerdahl and Jackendoff [7]). In general, the listener uses tonal-temporal hierarchies to combine notes to form patterns. Other important efforts to characterize wholeness in musical sequences are represented by Meyer [8], Huron [9], Temper [10] (theories of expectation); Lerdahl and Jackendoff [7] (generative theory of tonal music); Narmour [11](implication-realization theory); and Margulis [12] (musical tension models). Ultimately, all these mechanisms are Gestalt- and culture-based mechanisms. For an excellent account on these perceptual mechanisms, see Oxenham [13] as well as Deutsch [14] in the book *The Psychology of Music.*

The other important feature in the definition of musical pattern is repetition. How often and where in the piece a pattern is repeated definitely shapes the perception by the listener. This is what Nattiez [15] (and also Lartillot [16]) calls syntagmatic relations, which just refer to the syntax of the pattern in the piece. By musical syntax here we refer to the order in which musical elements appear in a given piece. Margulis [17], one of the few authors who has examined musical repetition in depth, states that, "music's repetitiveness is at once entirely ordinary and entirely mysterious." Repetition in language as it occurs in music would be deemed unacceptable. However, repetition in music is key as it greatly contributes to creating meaning and structure. Repetition also plays an important role in creating, denying or delaying musical expectations, which is a mechanism to produce musical meaning. Margulis [17] highlights three primary roles in repetition in music: (1) learning and level-shifting, (2) segmentation, and (3) expectation. It should be noted at this point that by repetitions we do not mean literal repetitions. Two sequences that bear certain similarities can be considered as the same sequence; for example, a sequence transposed by a fixed interval results in a similar sequence to the original one. Other perceptual pattern-preserving operations are changes of voice, change of tempo, and minor changes of duration, contour, or pitch. The extent to which two patterns can be considered the same strongly depends on the music style under study. Rolland [18] terms this relation between patterns equipollence (he claims it is more general than a similarity relation).

Notwithstanding the fact that rhetorical patterns may present musical wholeness and a certain degree of structural repetition on their own, their musical meaning stems from extramusical reasons, provoking emotions and conveying textual meanings in the musical discourse being the main ones. Therefore, we only have to look for repetitions as musical wholeness is given by the very nature of the rhetorical pattern.

Determining the importance of a musical pattern is also a significant issue in the computational analysis of music data, since the mining of frequent patterns leads to combinatorial problems [19], [20]. Closed patterns provide a succinct and robust method of reducing the existent redundancy of the set of frequent patterns by discovering a small subset of it. A pattern is said to be closed if it is not included in a larger pattern that occurs with the same frequency. As we can see by this simple definition, closed patterns reduce the search space of frequent patterns by providing at the same time a more compact representation. In this sense, [21] evaluates sequential pattern mining on a corpus of Mozarabic chant neume sequences. Their results indicate that it is possible to retrieve all known patterns with an acceptable precision using significant closed pattern discovery. In musical terms, and more specifically in this work, a closed pattern will help determine patterns that are associated with rhetorical figures.

B. Rhetorical Patterns

Among the many definitions of rhetoric in language, a very concise one is provided by Quintilian [22], who states that "rhetoric is the science of speaking well, [...] the science of correct expression." From this definition, the classical authors established three purposes of rhetoric, namely, to inform, to persuade or move, and to entertain or delight. According to the classical authors, rhetoric can be further divided as follows [23]: from the composer standpoint, invention (*inventio*), arrangement (*dispositio*), elaboration and decoration (*elocutio*); from the performer standpoint, delivery (*pronunciatio*), memory (*memoria*), and gesture (*actio*). The *Harvard Dictionary of Music* in turn defines rhetoric in music [24] as follows.

> The principles governing the invention, arrangement, and elaboration of ideas in a piece of music. Drawing on classical models of oration, music

> theorists cultivated the concept of musical rhetoric in earnest during the 16th, 17th, and 18th centuries, especially for works with texts. Such activity blossomed into the so-called doctrine of figures and doctrine of affections. Although rhetorical models for music were supplanted in the 19th century, they continue to influence the various modes of musical analysis, whether or not the music in question is based on an explicit text or program.

Although similarities between both definitions can be noted, the relationship between rhetoric in language and rhetoric in music is a very complicated issue. The very analogy between music and language is already a delicate matter. We find many sources in the history of music that shed light on this connection. In the German context, theorists such as Calvisius, Lippius, Nucius, Thuringus, Kircher or Mathesson are studied and mentioned in several rhetorical analyses in contemporary sources [23], [25], [26]. The reason for focusing on German authors is their penchant for theorizing rhetorical figures of speech, contrary to what happens in Italy, where theorists do not explain these figures and just apply them in their musical compositions. During the 16th century, the discovery and absorption of the ideas in the classical rhetoric texts resulted in a new expresiveness that permeated the language and all the arts, including music. The influence of rhetoric brought about a new consciousness of musical expression. The rhetoric discourse in music has been especially important between the 16th and 18th centuries. Furthermore, in this research we mainly focus our attention on imitative figures. Most figures in musical rhetoric are divided into those of development by repetition, comparison or amplification, and surprise [23]. After combing through the relevant literature, we chose to follow López Cano [26] and Bartel [25] as our sources for defining the rhetorical figures included in this study. From a systematic musicology standpoint, little attention has been devoted to rhetorical patterns in the past few years. A notable exception is Parada-Cabaleiro [27], who studied *madrigalisms*, a composition technique that mimics the linguistic content of the lyrics.

In general, in order to determine if two sequences are the same, rhythmic and intervallic information have to be taken into account. Given the highly imitative nature of the rhetorical patterns, rhythmic information was not necessary to detect the rhetorical patterns in the corpus. In fact, due to the flexible use of the rhythm in the Renaissance period, it is very complicated to establish a rule to identify each different case (augmentations, diminutions, lengthening or shortening patterns). Therefore, only intervallic patterns will be considered here. Thus, two sequences will be considered the same if their generic-interval sequences are identical (this includes length, order, and contour of intervals in the pattern). By generic interval we refer to the interval category, e.g., third or sixth, without considering the interval quality, e.g., major or minor. That is, a minor third and a major third are the same generic interval of a third. This definition not only encompasses exact repetitions of a pattern, but also its transposed versions where modality can vary. Two additional constraints were placed to define and detect rhetorical properly:

1. The frequency of the repetition of a pattern has to be greater than 3 repetitions per work, in any voice.
2. The minimum pattern length considered has to be 3 notes. The selected figures of speech and their definitions are given below.

- **Epizeuxis**: an immediate and emphatic repetition of a pattern in any voice, which normally is short and insistent [25]. We added "short and insistent" to clarify the difference among other figures. If we draw an analogy with the speech, a repetition of a name would fit with the concept; i. e., *John, John, John, I need your help*. Since we always find this figure as a short pattern, usually in stretto, and very insistent in several voices, we have to lay down some criteria to properly identify this figure:
  (1) The maximum length of this figure is the equivalent to four onsets.
  (2) The maximum separation distance among the patterns will not be larger than half a bar. Otherwise, the insistence effect would be lost.
  (3) The patterns may or may not overlap in time.

  In the excerpt from the "Kyrie" of *Ave Maris Stella* mass shown in Fig. 1, we find an instance of an epizeuxis where a 3-note pattern is repeated throughout two voices and with less than half a bar apart between consecutive repetitions of the pattern. The two first patterns overlap in time; the second and the third do not.

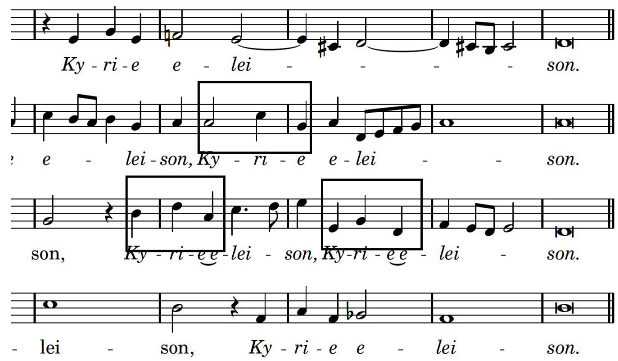

Fig. 1. Example of epizeuxis ("Kyrie" of *Ave Maris Stella*, bars 18-22).

- **Palilogy**: a repetition of a pattern transposed by an interval other than unison or octave in other voices [25]. Fig. 2 shows an instance of palilogy (beginning of the "Kyrie" of *Ave Maris Stella*).

Fig. 2. Example of palilogy ("Kyrie" of *Ave Maris Stella*, bars 1-4).

- **Synonymia**: repetition of a pattern transposed by some interval in the same voice [26]. In Fig. 3 on next page, we can see a synonymia, since the imitation coincides in the same voice in different pitches; in this example two synonymias are shown. The excerpt below belongs to the 4-voice "Kyrie" of *Missa pro Defunctis* mass.
- **Polyptoton**: repetition of a pattern in other voices either in unison or at the octave [26]. This author—following other authors such as Arnold Shering, Hans-Heinrich Unger, and George Buelow—only mentions unison. We expanded the term to the octave; in vocal music it is very common to transpose patterns an octave up or down to adapt them to the vocal range. As we can see in Fig. 4 on next page, the same pitches are replicated in another voice at a higher octave (notice the suboctave clef); therefore, we are dealing with a polyptoton. This example has been selected from the "Kyrie" of *Ave Maris Stella* mass.

Notice that in some cases a rhetorical pattern of one kind may contain patterns of other kinds as sub-patterns. For instance, a palilogy may contain other shorter palilogies that are more frequent than the palilogy itself. This will be termed as the sub-pattern problem and it will be discussed in the result section. Furthermore, the rhetorical categories epizeuxis and palilogy are not mutually exclusive. A palilogy may hold hold conditions (1) and (2) in the definition of epizeuxis and therefore it can in fact be an epizeuxis. This situation will arise in the results of the experiments as we will see later.

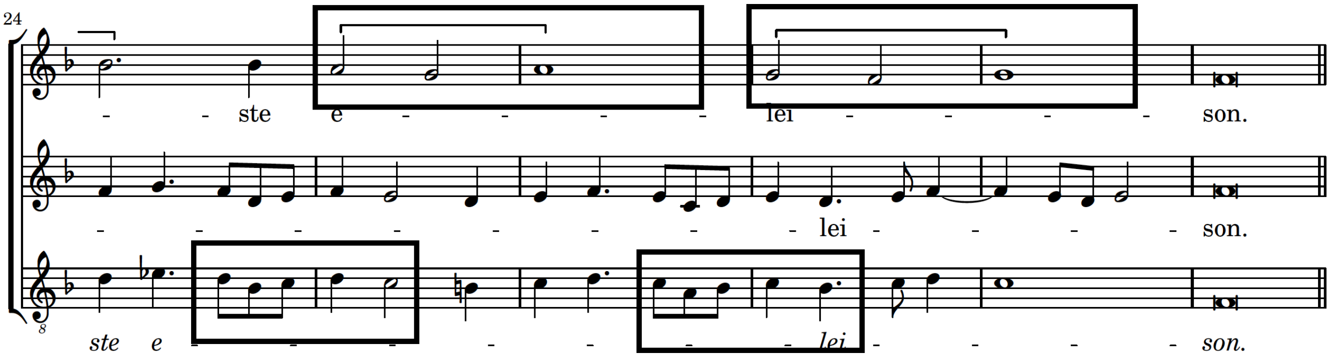

Fig. 3. Example of synonymia ("Kyrie" of Missa pro Defunctis mass, bars 24-28).

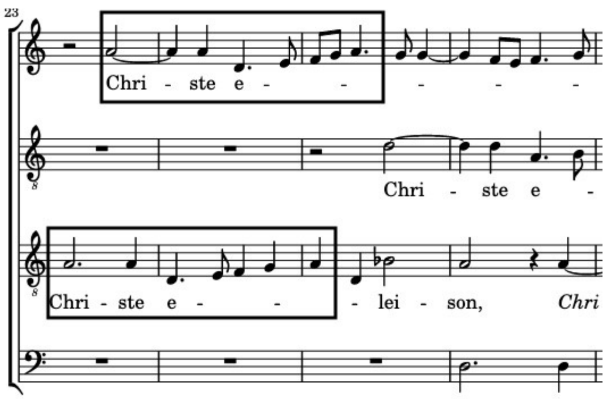

Fig. 4. Example of a polyptoton ("Kyrie" of Ave Maris Stella mass, bars 23-26).

C. A Musical Corpus of Renaissance Music

A corpus from the masses of Tomás Luis de Victoria (1548–1611) was chosen to analyze rhetorical patterns. We took 4-, 5-, and 6-voice masses in order to facilitate the data extraction process (more than six voices could have further complicated the process). Victoria is one of the main representative composers in the music of the Spanish Golden Age. With the advent of the Protestant Reformation, a Counter-Reformation is born, the Council of Trent being a pivotal event. This ecumenical council will have an important implication in music composition. Victoria's music perfectly reflects the tenets of this Council, whose musical characteristics are the following:

- The intelligible presentation of lyrics;
- The elimination of profane elements;
- Uncomplicated and understandable counterpoint, and hence, melodic lines that are streamlined and easy to remember.

As we can infer, musically speaking, text intelligibility was one of the most important issues in the Council of Trent. For that reason, the counterpoint is at the service of the text, the melodic lines are very clear, text is understandable, and the imitation is very balanced.

In addition, Victoria studied in Rome, and as stated by Wagstaff [28], it is likely that he met Palestrina in that period (from 1563-65 to 1587). But although his music is clearly influenced by Palestrina's style, Victoria's compositions are very personal and underwent a constant evolution throughout his lifetime. For instance, his mass *Ave maris stella* displays concise musical ideas, which fits with the spirit of the Council of Trent. By way of contrast, the mass *Salve regina* uses compositional procedures close to music of Giovanni Gabrieli or Monteverdi [28].

Victoria's music is characterized by imitative structures in melodic lines. Many of his compositions fit within the most popular musical settings in the Renaissance: cantus firmus, paraphrase, and parody, which are based on elaborating new compositions from pre-existent material [29]. Examples of these procedures in his works and in other Spanish composers (Francisco Guerrero and Cristobal de Morales) can be perused in Stevenson [30].

In connection with this paper, Victoria's discourse fits in the rhetorical spirit of the Renaissance era. As a matter of fact, not all figures of speech can be found in his music, we have to think that the limitations of Catholic music in this period restricted many expressive procedures, such as the treatment of consonance and dissonance. We thus contend that repetitive figures are very consistent in Victoria's music, which was the main reason to choose him for our research.

As for the corpus used in this study, a selection of four masses was made taking into consideration their relevance in the composer's opus and the time span from the earliest composed work to the latest. The masses are *Dum Complerentur*, *Ascendens Christus*, *Gaudeamus*, and *Veni Sponsa Christi*. The transcriptions were made by musicologist Nancho Álvarez and the MIDI files were taken from his web page [31]. Nevertheless, his transcriptions were checked against the original manuscripts.

III. Rhetorical Pattern Finding

A. Musical Annotations and Assessment

The methodology presented here is of deductive nature, that is, the rhetorical patterns were defined on an abstract level. To test the goodness of our definition, we set up a classification task consisting of automatically searching for those patterns in the Victoria corpus. For both tasks —providing a conceptual definition of rhetorical patterns and find them in the corpus— careful assessment was required. In order to deal with both issues, we had a small set of experts at our disposal. Five experts with different musical background, from musicologists or composers to performers, formed the group of annotators. Their task was to look at the four Victoria's masses and extract the rhetorical patterns by following the definitions provided in Section II.B. The patterns extracted by the experts would be compared to those extracted by the BIDE algorithm. As a matter of fact, not all the experts agreed upon the annotations and much heated and in-depth discussion broke out. Part of the objections raised by the experts led to a refinement of both our definition of rhetorical patterns and the computational model. A delicate issue was how to deal with the consensus-reaching problem. In order to solve that problem, we employed the Delphi method.

The Delphi method has been extensively used in several fields of health research [32] and hospitality sector [33]. This method is a systematic way to achieve consensus by a group of experts in subjects where the results are not so obvious (as it is our case). In fact, expert consensus is the ground of science in important decisions such as funding applications, publications, or different metrics of citation. Consensus methods are mainly based on the idea contained in James Surowiecki's book *The Wisdom of Crowds* [34]. This author presents a few ideas to successfully devise a consensus-reaching scheme:

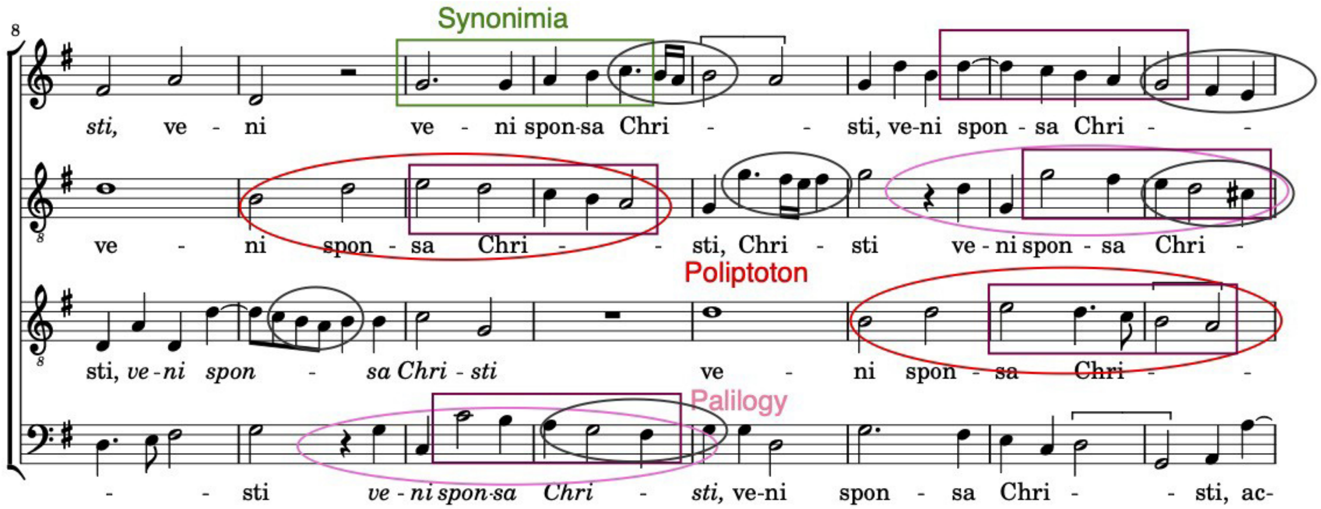

Fig. 5. Annotations of the rhetorical patterns by the experts.

TABLE I. Delphi Rounds for Consensus-Reaching

Delphi rounds	Rhetorical pattern model	Ground truth
Round 1	• Model with many low-level features both in rhythm and pitch • Based on the idea of main note • Based on changes of melodic contour • Incorporated a melodic similarity measure • Some features were not computational	• The annotations presented marked levels of discrepancy • Issues: closed patterns, overlapping patterns, different definitions of rhetorical patterns in the sources, selection of the final patterns • Experts selected patterns, but were not able to formalize their choice
Round 2	• The model is simplified, but still is low-level oriented • Still based on melodic contour and similarity measure	• Fewer differences in annotations than in the first round • Consensus on the sources and definitions grow • The issues of closed patterns and overlapping patterns persist
Round 3	• Low-level oriented model is abandoned in favor • of a simpler model • Closed patterns with constraints are introduced	• Consensus on the sources and definitions is reached • Annotators justify better their choices • Differences in annotations persist in a few cases
Round 4	• The model is only • based on intervallic content • Rhythm is not taken into account • The model is fully computational	• Very high degree of consensus on the annotations • Realization that intervallic content suffices for this study

diversity of expertise, independence of experts, de-centralized work, and a mechanism for aggregating different ideas. The Delphi method works as follows: (1) A person (the facilitator) organizes the study and recruits some experts; (2) The facilitator prepares a questionnaire and collects the responses from the group of experts; (3) The facilitator provides the experts with anonymous feedback and they review their responses; (4) After a few rounds, normally 3 or 4, the group of experts arrives at a reasoned consensus. The Delphi method has started to be used in areas other than health research. Romero [35] used it to agree upon a definition of a good musician among different experts. In education it is also taking on; see Green [36] for more information.

As we pointed out earlier, in our work, the Delphi method had two uses: (1) gather knowledge and insight from the experts to build the formal definitions of rhetorical patterns; (2) to build a ground truth for validating the pattern-finding algorithm used in our experiments. The number of Delphi rounds carried out was four. One of the authors of this paper acted as the facilitator. The experts annotated the scores in staff notation through a color and number system to keep track of the patterns. After annotating the scores, the experts would write a report where they pointed out to discrepancies or issues found during the process. In Fig. 5, an excerpt of a score annotated by an expert is shown. As the reader can see in the figure, the rhetorical patterns can overlap.

There were two main lines of inquiry at the Delphi rounds, namely, development of the rhetorical pattern model and establishing the ground truth. The main issues encountered at the beginning of the Delphi rounds were: the model was based on low-level features that in many cases could not be described in computational terms; there were discrepancies among the experts in the definition of the rhetorical patterns; experts experienced difficulties at formalizing their annotations; it was not clear how to deal with overlapping patterns. During the rounds, these issues were resolving gradually. Closed patterns were incorporated to the model (the initial definition of pattern was not fully versatile), the equality between sequences was defined just in terms of their generic intervallic content, and the model was improved and the experts reached a high level of consensus. The resulting model was presented to the annotators and discussed with them. As a matter of fact, it was a parallel process. Another issue concerning the assessment of the algorithm was the number of pieces involved. Although four masses can seem a small number, it turns out that the number of patterns within them was high enough for our purposes. This issue was also discussed with the experts, who agreed that the number of masses was adequate. In the results section the annotations by experts are compared with the patterns mined by the algorithm[1]. Table I presents a summary of the four rounds carried out and the main points under discussion. Referring to the table, a main

[1] The reader can find an annotation of one of the sections of Victoria's masses in https://www.dropbox.com/s/ysmuc3im34rqylo/Rhetorical-patterns-Annot.pdf?dl=0. This was one of the annotations used as ground truth in this study.

note is a note that is more prominent that others, which can be due to melodic, rhythmic or harmonic reasons, among others. By very high degree of consensus, here we mean that at least four experts agreed on the annotations. However, any source of disagreement was analyzed and discussed by the experts and often there was unanimity.

B. Computational Rhetorical Pattern Finding

As stated at the outset, in this section will consider the problem of building a computational model for finding rhetorical patterns. A computational model is a description of a phenomenon, in our case rhetorical pattens, given in terms understandable by a computer. The first component of our model consists of the encoding of the musical piece in the corpus. The corpus was encoded by employing a string representation of intervals given in the software Music21 [37], and the chromatic pitch values from the MIDI files of the pieces. Each voice is divided into phrases by rests.

The second component is composed of the computational description of the definitions of rhetorical patterns given in Section II.B. Notice that all the features of the rhetorical patterns described in there can be expressed in computational terms. The third component is the concept of closed pattern, which allows eÿcient pattern-discovery in musical databases.

To computationally determine the rhetorical figure associated with a pattern P, we establish four categories that a pattern can belong to: epizeuxis (Ep), palilogy (Pa), synonymia (Sy), and polyptoton (Po). Recall that a pattern is a sequence whose frequency of occurrence is equal or greater than 3 and whose length is at least 3 notes. Their definitions are as follows:

- A pattern $P \in Ep$ if and only if there exists another sequence P' in any voice such that P and P' hold the following conditions: (1) P and P' have the same intervallic content; (2) the length of P is not greater than four onsets; (3) the maximum separation between P and P' is not greater than half a bar.
- A pattern $P \in Pa$ if and only if there exists another sequence P' in a different voice transposed by some interval other than the unison or the octave such that P and P' have the same intervallic content.
- A pattern $P \in Sy$ if and only if there exists another sequence P' in the same voice transposed by some interval other than the unison and the octave such that P and P' have the same intervallic content.
- A pattern $P \in Po$ if and only if there exists another sequence P' in a different voice transposed to an interval of unison or octave such that P and P' have the same intervallic content.

It could be expected that the rhetorical categories were disjoint. However, they are not due to the broad definition of the epizeuxis. Putting aside the length and separation of the patterns, in an epizeuxis, the patterns can be transposed by any interval and appear in any voice. This causes that palilogies, synonymias, and polyptotons may also be epizeuxis on certain occasions.

From the corpus of symbolic music encoded by using the string representation defined in this sub-section, we develop a method to obtain rhetorical patterns following the closed constrained patterns with the above-described definitions; such method is based on the BIDE algorithm [38] and a filtering mechanism. The BIDE algorithm is an eÿcient algorithm for mining frequent closed sequences without candidate maintenance. It prunes the search space more deeply compared to previous algorithms; see [38] and the references therein. The input parameters for the BIDE algorithm are the set of sequences and the minimum support threshold (minimum frequency of a pattern).

Our method works as follows:

- Given as input the set of intervallic sequences and a minimum support threshold, we obtain as output the set of all closed patterns **C** by applying the BIDE algorithm to the input.
- For each pattern in **C**, we test whether that pattern meets any of the rhetorical definitions and assign it to its corresponding categories. We thus obtain as output the set of all closed patterns that meet the rhetorical constraints.

For the sake of reproducibility of our experiments, the code is available at OMITTED FOR BLIND REVIEW.

C. Experiments

Our experiments consisted of running the BIDE algorithm on the corpus and assigning the closed patterns output by the algorithm to the rhetorical categories. This assignment was done by applying the formal computational definitions given in Section III.B. Table II lists the number of rhetorical patterns by category extracted by the experts and by the computational method proposed. In the case of palilogy and epizeuxis —since these categories are not mutually exclusive—, there were 8 patterns classed as both palilogy and epizeuxis. Those 8 patterns were not annotated by the experts. These patterns are all longer patterns that contained shorter patterns annotated by the experts.

TABLE II. Total Number of Rhetorical Patterns Extracted By Experts and by the Computational Approach Presented

	Experts	Algorithm
Palilogy	54	75
Epizeuxis	12	32
Synonymia	5	5
Polyptoton	35	33
Sum	106	145

Taking a closer look at Table II, we can see that the rhetorical figures obtained by the experts and the algorithm are exactly the same in the case of the synonymia. In the case of polyptoton, the experts found two additional patterns than the algorithm did not. Epizeuxis and palilogy show noticeable differences in number. For these two categories, the algorithm found more patterns than those annotated by the experts. Differences in polyptotons are associated with smaller length rhetorical figures such as epizeuxis, that are detected by the algorithm as closed patterns and selected in favor of longer polyptotons. This is indeed something that may require further discussion.

Closed patterns with constraints may capture sub-patterns that have a frequency of occurrence higher than the super-patterns that contains them. In the context of rhetorical figures such a definition does not always work. For instance, a palilogy may contain closed patterns that are more frequent than the palilogy itself, and in other type of musical analysis those closed patterns may be considered important motifs within a larger musical sequence or phrase. In the analysis of rhetorical patterns, such closed patterns that are incorrectly identified as epizeuxis, mostly because of the pattern length and how they overlap with each other. That is another reason to see sharp differences in the number of epizeuxis annotated by the experts and the ones discovered by the method proposed.

IV. Results

We defined a classification task by taking the annotations made by the experts as a gold standard. If a pattern was classed as in the annotation by the experts, then we considered the pattern as a correctly classified pattern. In Table III, metrics to measure the performance of the classification task are presented for both all the rhetorical patterns and by pattern individually. We denote by tp, tn, fp, and fn, true positives, true negatives, false positives, and false negatives, respectively. True positives are the annotated patterns

TABLE III. Overall Metrics and by Rhetorical Pattern

Metrics	Overall	Palilogy	Epizeuxis	Polyptoton	Synonymia
True positives tp	104	54	12	33	5
False positives fp	41	21	20	0	0
False negatives fn	2	0	0	2	0
Precision (%) = $\frac{tp}{tp+fp}$	71.73	72	37.5	100	100
Recall (%) = $\frac{tp}{tp+fn}$	98.11	100	100	94.28	100
F -score (%) = $2 \cdot \frac{Precision \cdot Recall}{Precision + Recall}$	82.87	83.72	54.54	97.05	100

correctly classified by the algorithm. Given the nature of this classification task, because the algorithm does not actually reject non-rhetorical patterns, there are no true negatives and, therefore, $tn = 0$ in all cases. False positives are those patterns identified as rhetorical patterns by the algorithm that were not annotated by the experts. Finally, false negatives are those patterns annotated by the expert that the algorithm failed to identify. In the last three rows, three metrics to assess the performance of the task were computed, namely, precision (positive predictive value), recall (true positive rate), and *F*-score.

Due to the high number of false positives in the case of palilogy and epizeuxis, precision is relatively low, especially for epizeuxis where there are many more false positives than true positives. Since there are no false negatives except for polyptoton, recall is 100%. For the polyptoton, the algorithm failed to find two annotated patterns and then recall is not 100%. The values for *F*-score are high, except in the case of epizeuxis, again due to the fact that the number of false positives was relatively high compared to the number of true positives.

An important type of implicit contextual knowledge that experts have and that the method presented in this article has problems with is identifying the palilogy. There are several instances where the algorithm labels patterns as palilogy and experts do not consider those patterns as rhetorical. These cases are related to the time of occurrence between two such patterns, that is not always clearly identifiable in a quantitative or constraint-based way. Experts tend to pay close attention to the development of musical phrases and detect rhetorical implications of patterns based on how music develops within a musical work. Context and the awareness of musical development plays an important role in filtering out such patterns by the experts.

V. Conclusions

In this article an initial investigation of rhetorical patterns in the music of Tomas Luis de Victoria was presented. A conceptual framework for rhetorical patterns was established, which in turn lead to the computational model of aforesaid rhetorical patterns. One of the main goals of this study is to test out the computational model. In order to do so, we intentionally chose a relatively small-size corpus of highly imitative music such as Victoria's masses. Annotators were asked to categorize patterns found by pattern discovery and this was compared to the categorization assigned by the computational model. It was shown that the closed-pattern mining approach with constraints produces good results that can be easily explained in musical terms. Furthermore, this approach can be a useful aid to the musicologist in the discovery of relevant rhetorical patterns. Actually, our future research will consist of designing a computer-aided system to study rhetorical patterns, which will include looking for them by using the computational models described in this paper. As seen in the results section, the computational model was robust enough to assign the same class as that assigned by the annotators for most of the patterns and discovered some other additional patterns, which were obtained mainly due to the sub-pattern problem described above. Moreover, a novel approach to reaching consensus among experts, the Delphi method, was used. This method improved inter-annotator agreement.

However, several problems of contextual knowledge were detected in this study and future works should concentrate on how to incorporate that knowledge in the pattern discovery process. In our case, rhetorical patterns contained in larger rhetorical patterns posed delicate problems as how to interpret repetition within repetition and how to resolve ambiguity so that they are clearly classified as a unique rhetorical pattern. One potential approach to deal with this issues could be considering maximal closed patterns. Another possible approach to explore could be the polyphonic modeling of harmony and counterpoint in the pieces.

Since this was an initial study, the size of the corpus was kept intentionally small. As near future work, we intend to test out the computational model on larger corpora and gain more insight into rhetorical patterns in general and the sub-pattern problem. Considering other kinds of figures of speech, not only the imitative ones, is also an interesting avenue for further research.

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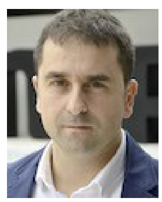

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